Overview

- Decision Making Under Uncertainty
- Probabilistic Constraints
- Optimization and Scenario Analysis
- Multidimensional Scenario Trees
- Stochastic Optimization

Decision making under Uncertainty

- How to deal with the uncertainty of achieving energy goals (e.g. 27% renewables by 2030; 20% energy efficiency by 2030)?
- How does the imposition of a greenhouse gas goal (e.g. 40% reduction by 2030) affect this goal?
- How to decide on policy instruments (e.g. carbon tax vs cap-and-trade) to achieve the goals?
- How to deal with the inherent uncertainty about technology and economy?

Decision making under Uncertainty

- How to evaluate the risks associated with pairing of electric energy from intermittent sources to fossil-based ones ?
- How to balance storage of energy from the grid in electric vehicles in the presence of variability of power supply and demand, volatile prices, and uncertainty in usage patterns for vehicles?
- How to decide on the load shedding of customers under unexpected system failure or supply insufficiency ?
- How to modify grid operators' unit commitment models to capture more renewable energy ?
- What is the best timing for irreversible power generation investments under uncertain price and demand levels ?

Probabilistic Constraints

Consider playing backgammon with its two dices. Dice one gives a result a_1 when thrown and dice two a result a_2. Consider a simple LP with two variables and one constraint:

$$\begin{aligned} \text{min} \quad & 5x + 6y \\ \text{s.t.} \quad & a_1 x + a_2 y \geq 3 \\ & x, y \geq 0 \end{aligned}$$

where
a_1=i (i=1,...,6) with probability 1/6
a_2=j (j=1,...,6) with probability 1/6

What does this LP mean ?

Ref. OR notes JE Beasley

Probabilistic Constraints

$$\begin{aligned} \text{min} \quad & 5x + 6y \\ \text{s.t.} \quad & a_1 x + a_2 y \geq 3 \\ & x, y \geq 0 \end{aligned}$$

where
a_1=i (i=1,...,6) with probability 1/6
a_2=j (j=1,...,6) with probability 1/6

One interpretation could be that we wish the constraint $a_1x + a_2y >= 3$ to hold for all possible values of a_1 and a_2. Then we simply have a deterministic LP with two variables and 36 constraints, i.e.

$$\begin{aligned} \text{min} \quad & 5x + 6y \\ \text{s.t.} \quad & ix + jy \geq 3 \qquad i=1,\dots 6\,;\, j=1,\dots 6 \\ & x, y \geq 0 \end{aligned}$$

Ref. OR notes JE Beasley

Probabilistic Constraints

Suppose now that instead of insisting that the constraint $a_1x + a_2y >= 3$ holds for all possible values of a_1 and a_2 we insist that it holds only with a specified probability $1 - \alpha$ (where $0 < \alpha < 1$).

For example $\alpha = 0.05$ would mean that we want the constraint $a_1x + a_2y >= 3$ to hold with probability *0.95*. Note the introduction of probability here: a constraint need not always be true now, rather it need only be true 95% of the time.

Hence the problem is:

$$\begin{aligned} \text{min} \quad & 5x + 6y \\ \text{s.t.} \quad & \text{Prob}(a_1x + a_2y \geq 3) \geq 1 - \alpha \\ & x, y \geq 0 \end{aligned}$$

Ref. OR notes JE Beasley

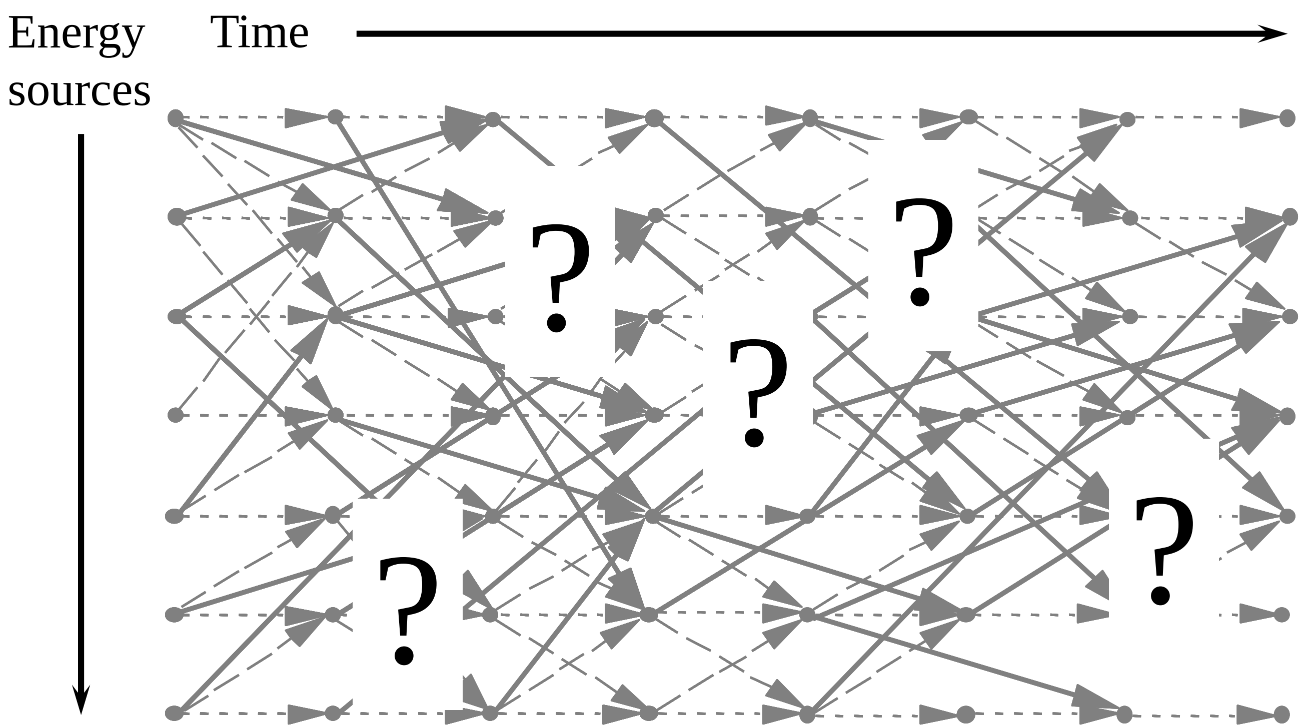
Probabilistic Constraints

The problem discussed above is an example of a *stochastic (linear) program with probabilistic constraints*. Such problems are also sometimes called *chance-constrained linear programs*.

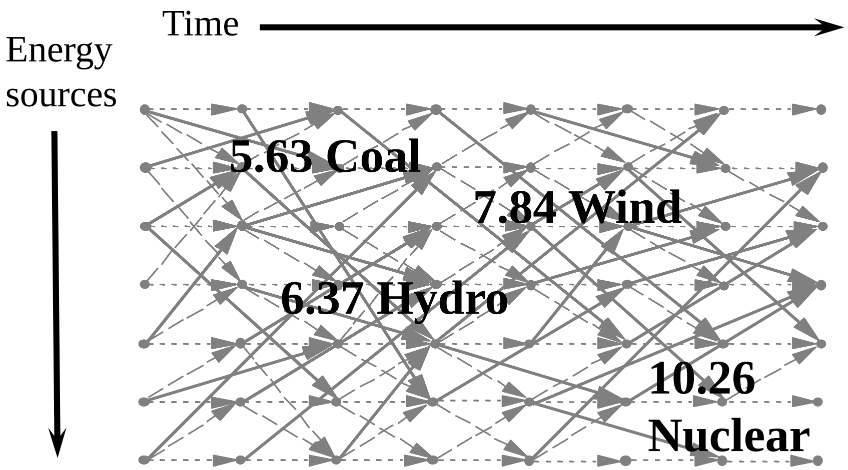
- ✓ GAMS implementation of a stochastic LP with probabilistic constraints

Optimization and Scenario Analysis

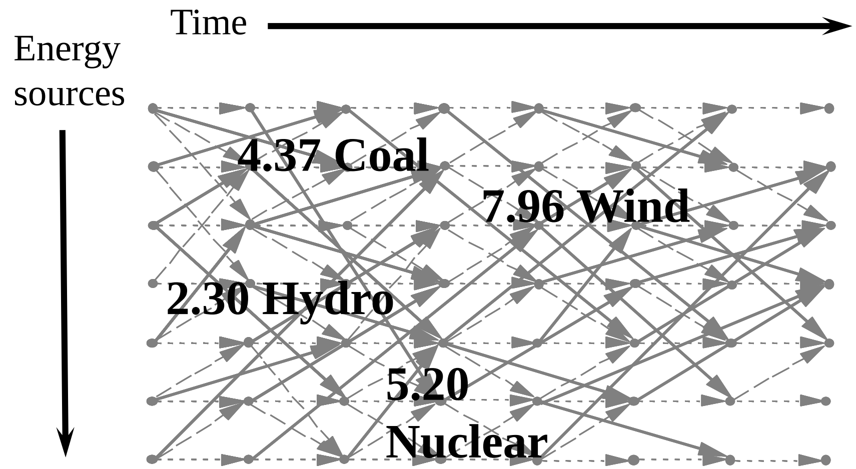
- Large-scale optimization (e.g. MARKAL, TIMES…)



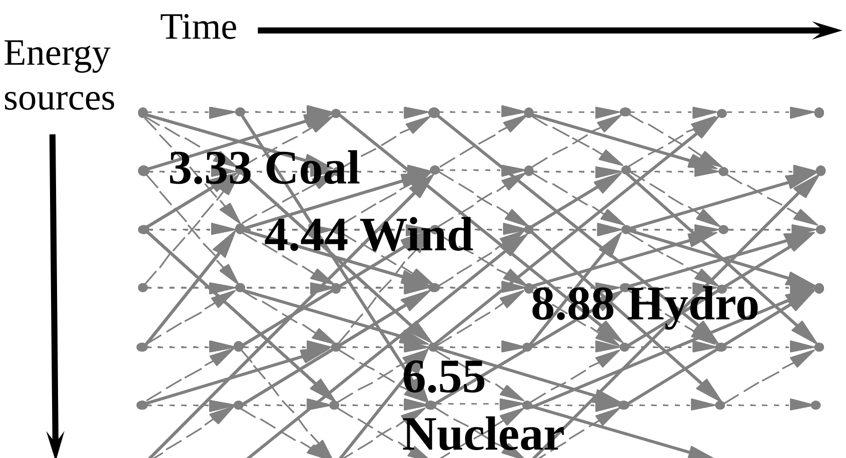
Multi-Stage Stochastic Programming in Energy



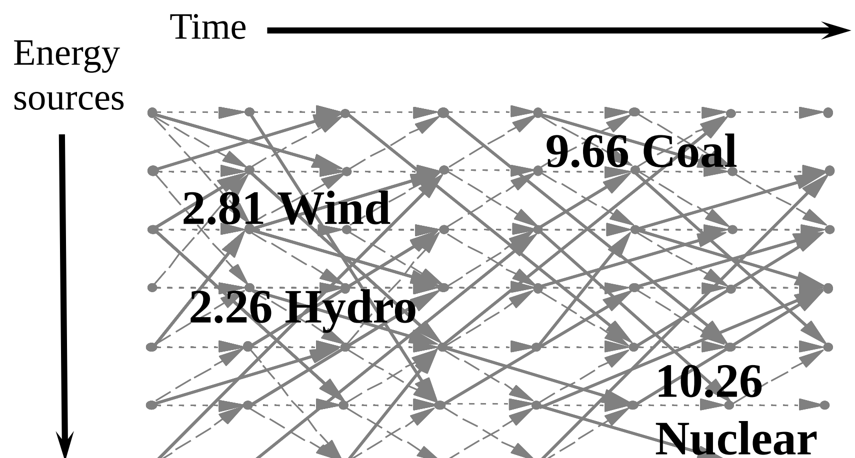
Scenario 1



Scenario 2

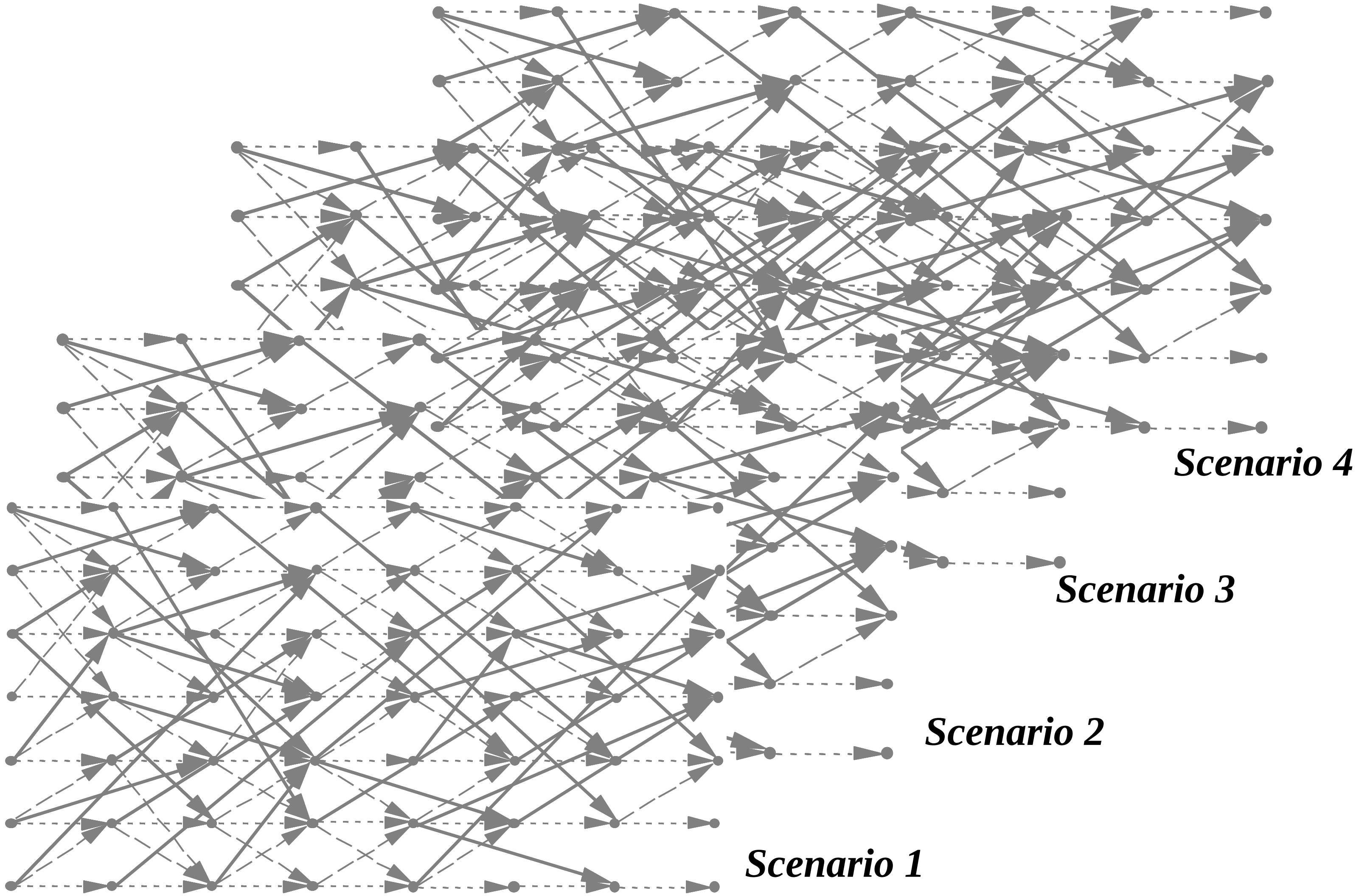


Scenario 3



Scenario 4

Optimization and Scenario Analysis



We have to combine the results of the scenarios to make decisions

Multidimensional Scenario Trees

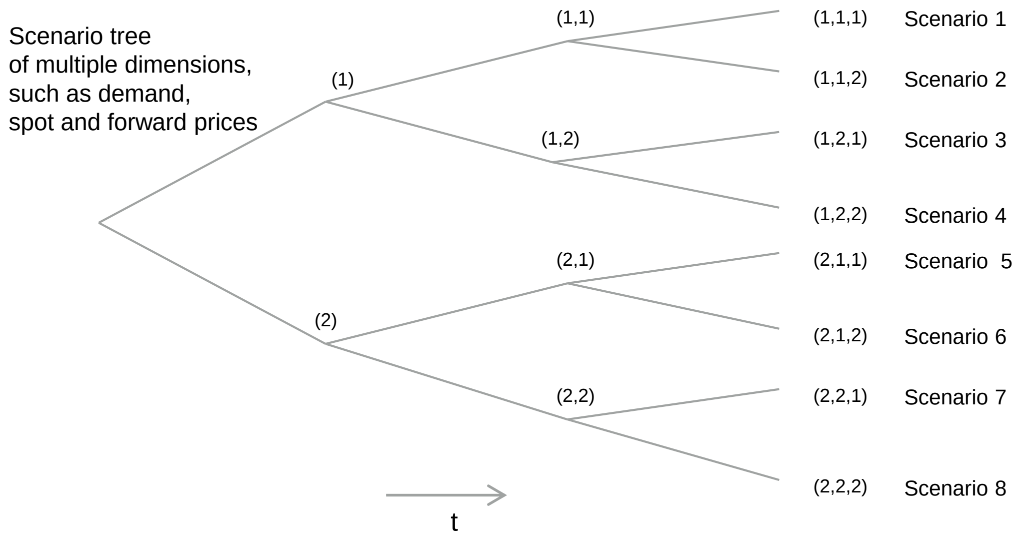
- Rather than generating multiple separate scenarios, stochastic optimization is based on the generation of scenario trees
  - Each single node of the scenario tree holds the future states of important uncertain impacts, such as
    - ✓ *spot market prices of electricity*
    - ✓ *forward market prices of electricity*
    - ✓ *oil & gas prices*
    - ✓ *demand levels*
    - ✓ *intermittent power generation levels*

Multidimensional Scenario Trees

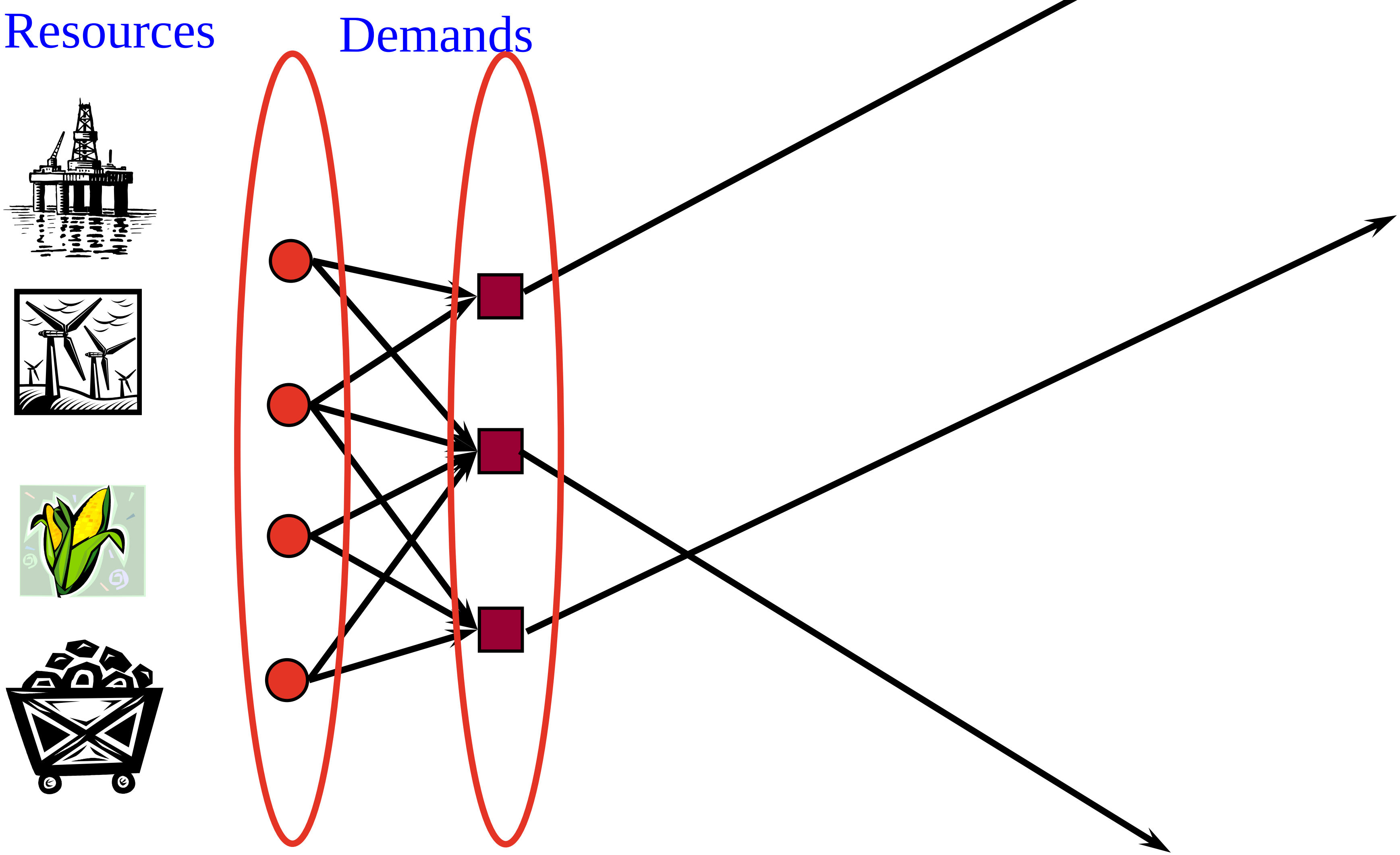
- Each path of the scenario tree represents a joint discrete evolution of all uncertain impacts, which altogether result in a multidimensional scenario tree. The values for all the multidimensional nodes are tpically calculated based on
  - expected value
  - evolution dynamics
  - volatility
  - correlation
  - stochastic processes (e.g. GBM, mean reversion)

Multidimensional Scenario Trees

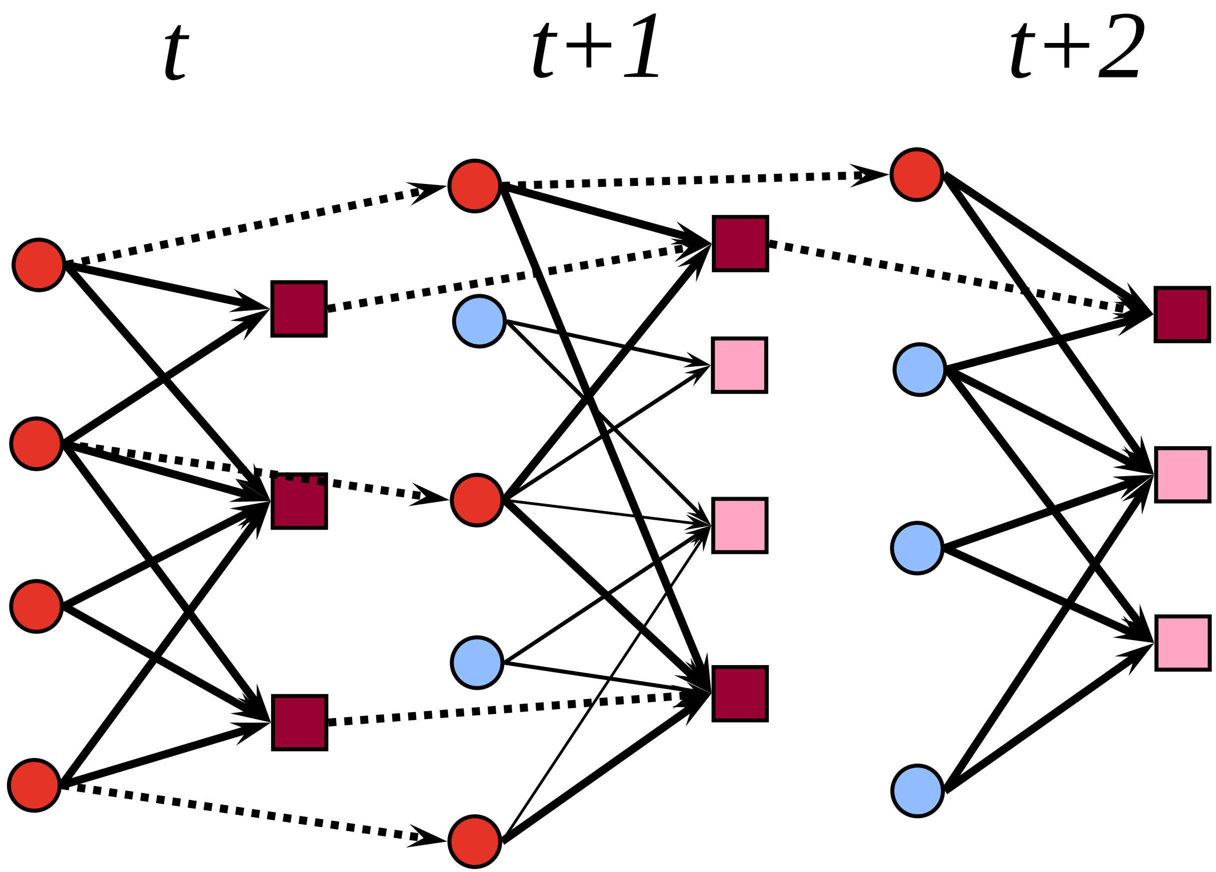
Stochastic Optimization:
Unique optimal decision in every node
with respect to all possible future developments



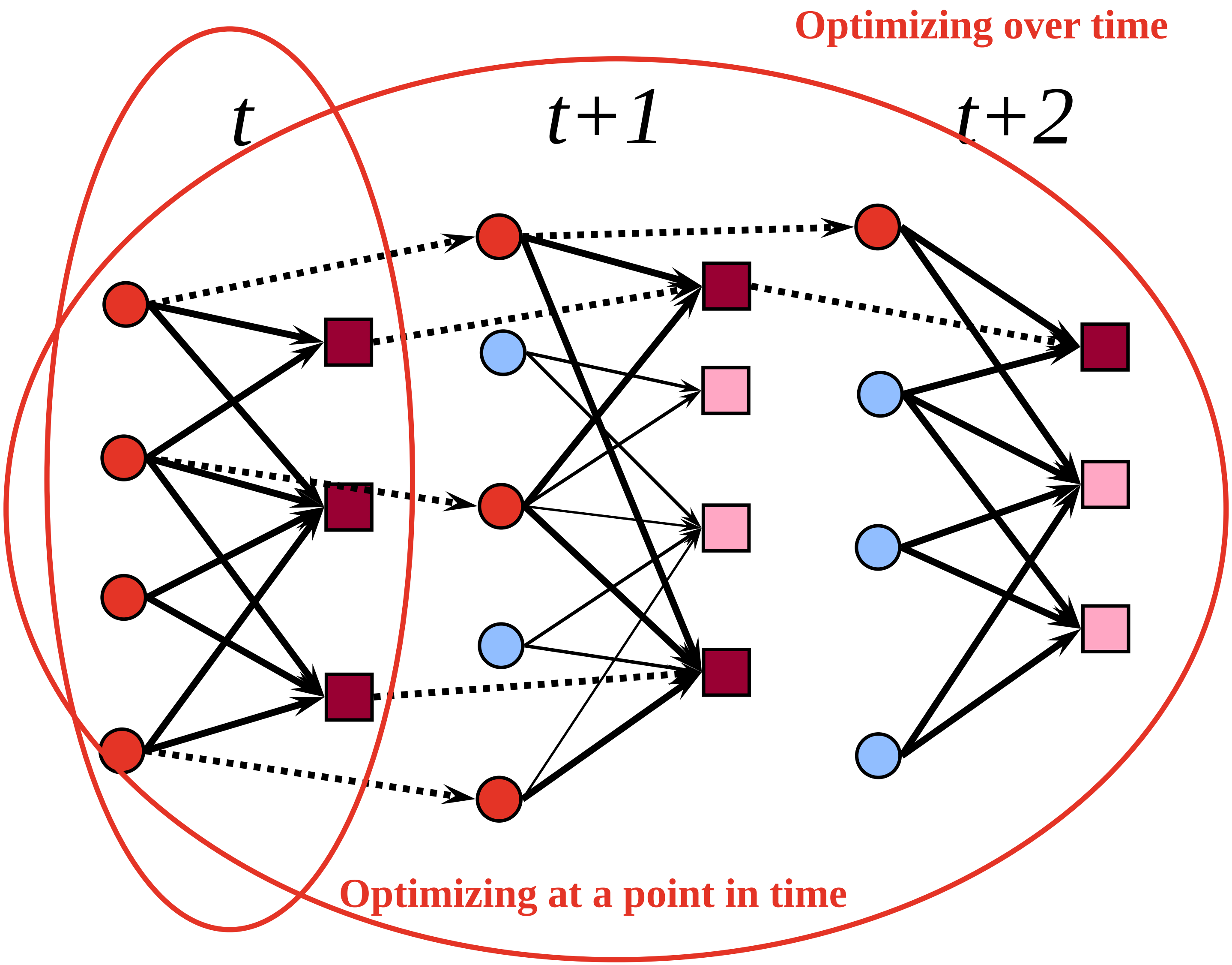
Stochastic Optimization



Stochastic Optimization



Stochastic Optimization



Two-Stage Stochastic Programs: Mathematical Formulation

Let $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ be two variables and let the set of all realizations of the unknown data be given by Ω, $\Omega = \{\omega_1, \ldots, \omega_S\} \subseteq \mathbb{R}^r$, where r is the number of the random variables representing the uncertain parameters. Then the stochastic program is given by

$$
\begin{array}{llllllll}
\text{Min}_x & z = & c^T x & + & \mathbb{E}[Q(x,\omega)] & & & \\
\text{s.t.} & & Ax & & & = & b, & x \geq 0, \\
& & & & & & & \\
\text{where} \quad Q(x,\omega) = & \text{Min}_y & & & q_\omega^T y(\omega) & & & \\
& \text{s.t.} & T_\omega x & + & W_\omega y(\omega) & = & h_\omega, & y(\omega) \geq 0, \quad \forall \omega \in \Omega.
\end{array}
$$

Stochastic Electric Power Expansion Planning Problem

APLP1: a two-stage stochastic Electric Power Expansion Planning LP. Facing uncertain demand, decisions about generation capacity need to be made.

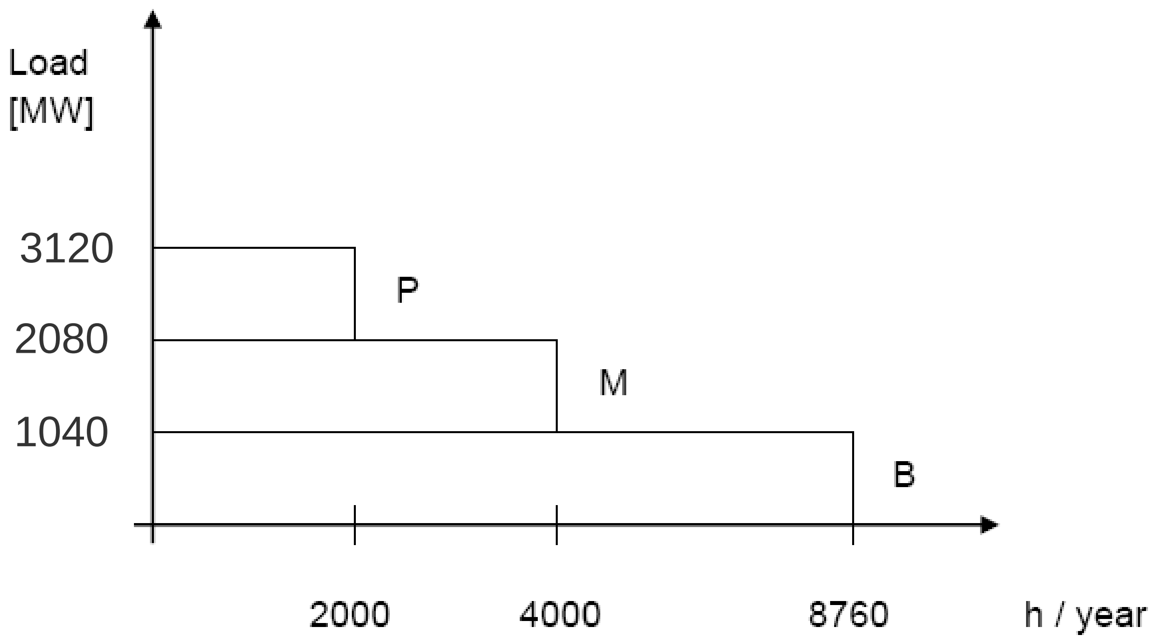
$$\min \sum_{j=1}^{2} c_j x_j + E\{\sum_{j=1}^{2}\sum_{i=1}^{3} f_{ij} y_{ij} + \sum_{i=1}^{3} f_{is} y_{is}\}$$

$$\begin{array}{llllll}
\text{s/t} & x_j & & & \geq b_j & j = 1,2 \\
 & -\alpha_j x_j + & \sum_{i=1}^{3} y_{ij} & & \leq 0 & j = 1,2 \\
 & & \sum_{j=1}^{2} y_{ij} + & y_{is} & \geq d_i & i = 1,2,3 \\
 & x_j, & y_{ij}, & y_{is} & \geq 0 & j = 1,2 \\
 & & & & & i = 1,2,3
\end{array}$$

d_i	demands in different load segments	f_{is}	penalty cost (VOLL)
α_j	availability of generator j	x_j	generator capacities
c_j	per-unit cost to build generator j	y_{ij}	operating level for generator j in load level i
f_{ij}	operating cost	y_{is}	unserved demand in load level i

Stochastic Optimization

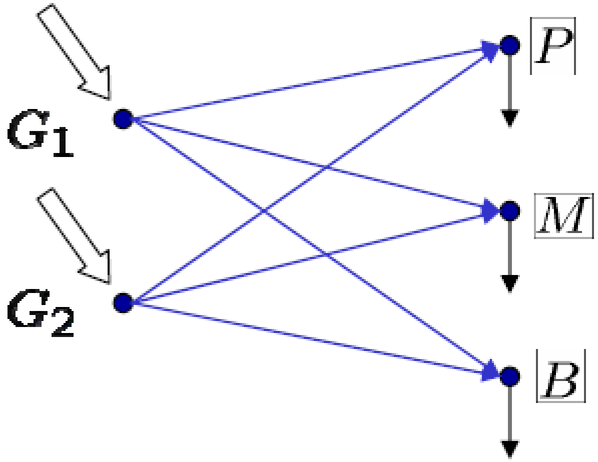
Electric Power Expansion Problem APLP1 (Infanger, 1994)



Load assumed to be concentrated in one node (no transmission network)

Stochastic Optimization

APLP1 BASICS

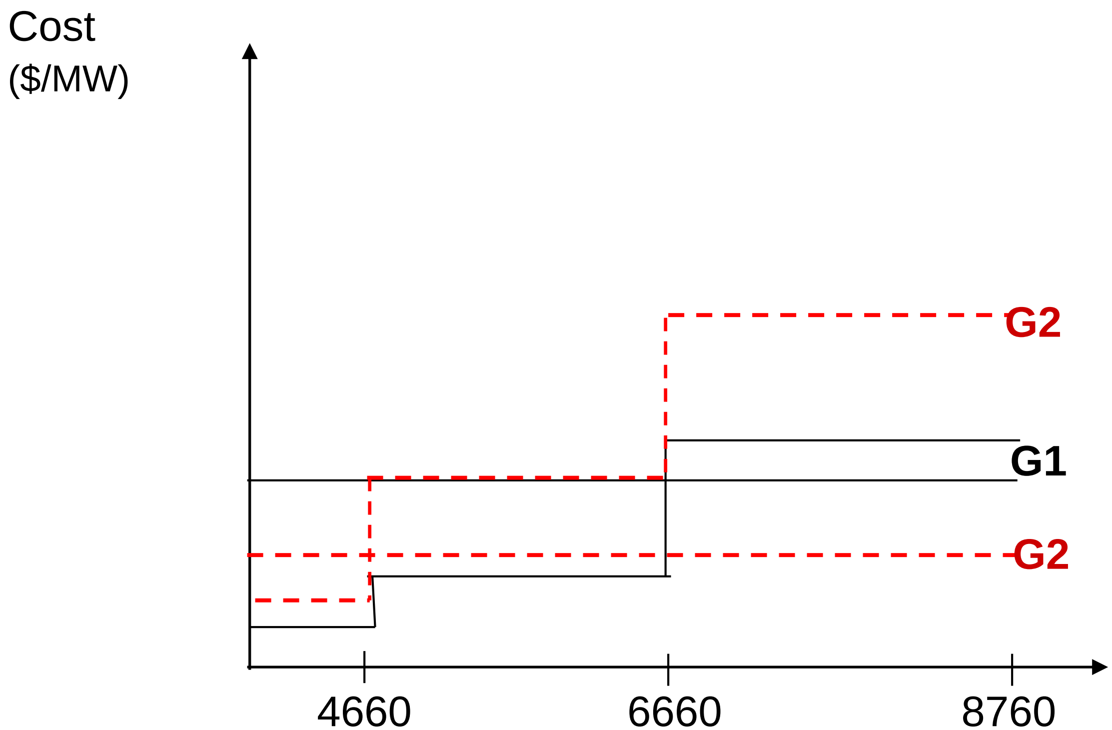


Two types of Generators : G1, G2
Three load segments : P, M, B

Model: Transportation network, no capacity constraints on the lines

Stochastic Optimization

APLP1 Capacity and Operating Cost



Stochastic Optimization

APLP1 Problem Data

Generator Capacity Costs (10^5\$/(MW, a))

$c_1 = 4.0, \quad c_2 = 2.5$

Generator Operating Costs (10^5\$/MW, a)

$f_{11} = 4.3 \quad f_{21} = 8.7$

$f_{12} = 2.0 \quad f_{22} = 4.0$

$f_{13} = 0.5 \quad f_{23} = 1.0$

Unserved Demand Penalties (10^5\$/MW, a)

$f_{1s} = f_{2s} = f_{3s} = 10.0$

Minimum Generator Capacities (MW)

$b_1 = b_2 = 1000$

Demands (MW)

#	1	2	3	4
Outcome	900	1000	1100	1200
Probability	0.15	0.45	0.25	0.15

Availabilities of Generators

Generator 1 (α_1)

#	1	2	3	4
Outcome	1.0	0.9	0.5	0.1
Probability	0.2	0.3	0.4	0.1

Generator 2 (α_2)

#	1	2	3	4	5
Outcome	1.0	0.9	0.7	0.1	0.0
Probability	0.1	0.2	0.5	0.1	0.1